Estimation of the solar radiation received by a solar collector

In this paper we will try to present the basic methodology for estimating solar radiation received by a collector, without making a complete statement of solar climatology.

Solar energy comes from thermonuclear reactions that occur within the sun, causing the emission of high power electromagnetic radiation, appearing much like a blackbody at 5,800 K.

Outside the atmosphere, the radiation received by the earth varies depending on time of the year between 1,350 and 1,450 W/m². It is then partially reflected and absorbed by the atmosphere, so that radiation received at ground level has a direct part I and a diffuse part D, the total G ranging from 200 W/m² (overcast) to about 1,000 W/m² (zenith clear sky). The energy received by a given surface depends on its tilt and orientation and local climatic conditions (figure 1).

Total irradiation G received by a solar collector can be decomposed into a direct component I cos(i) and a diffuse component D, which is often viewed as independent of the tilt, whereas strictly it is not. In some cases it is desirable to consider a direct component of radiation reflected by surfaces surrounding the collector. We then introduce the concept of albedo.

Solar radiation atlases at ground level are issued by national and international meteorological services in the form of maps and charts, on paper or computer. They are based on measures carried out daily in weather station networks, mainly for two values:

- sunshine duration, which is the period during which the direct radiation remains above a certain value internationally accepted. There are two readings per day, one for the morning, one for the afternoon. The unit used is the tenth of an hour of sunshine duration, and the measurement device is called a sunshine recorder;
- global horizontal radiation noted Gh, which is the energy received in a given time by a horizontal surface, from the sun and sky in the form of radiation of short wavelength. The hourly measurements are expressed in J/cm², the measuring device being called pyranometer.

To interpolate between the network stations, it is now common to use satellite data.

Furthermore, some stations are exceptionally equipped with pyranometer fitted with a visor strip that hides the direct component of radiation, thus allowing diffuse radiation to be measured.

The sunshine duration not being an energy measurement, to determine radiation received by a collector, there are usually only records of hourly global

---
1 For example for a collector facing south on a seafront or in front of a snowy field
horizontal radiation \(G_h\). It is therefore necessary to have methods as reliable as possible to separate direct and diffuse components of \(G_h\), methods generally based on statistical correlations validated from records of stations where \(G_h\) and \(D\) are measured simultaneously.

One both simple and reliable way is to correlate ratio \(D/G_h\) to \(G_h/G_0\), an indicator representing the ratio of global radiation received on Earth to global radiation outside the atmosphere, which has the great advantage of being determined on the sole basis of astronomical data. Such correlations (1) are called Liu and Jordan type, named after the scientists who have proposed them in the early 60s (for daily and not hourly values at that time).

\[
D/G_h = a \frac{G_h}{G_0} + b \tag{1}
\]

The values of \(a\) and \(b\) in equation (1) depend naturally on the location considered, and even the season if you want to be very accurate. Similar correlations can be established with other indicators, including the sunshine duration, but we will only present in detail in this section that corresponding to equation (1), and provide a small Excel macro allowing perform these calculations.

The establishment of astronomical equations requires the definition of two cartesian coordinate systems (Figure 2):

- equatorial coordinates, whose axes are given by the poles and the equator plane, the meridian of the place being taken as the origin. The daily astronomical tables of the sun are given in this reference;
- horizontal coordinates, whose axes are given by the vertical location and the horizon plane, the first meridian of the place being taken as the origin. It is the only coordinate system simple to use locally to measure the inclination and the solar orientation, as defined by angles \(\alpha\) and \(\delta\).

In Table 1, we find indicated successively notations used for angles 1 and 2 of Figure 2 in both coordinate systems.

<table>
<thead>
<tr>
<th>Table 11.1</th>
<th>equatorial coordinates</th>
<th>horizontal coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>Declination (D)</td>
<td>Altitude (h)</td>
</tr>
<tr>
<td></td>
<td>Hourly angle (Ah)</td>
<td>Azimuth (Az)</td>
</tr>
<tr>
<td>Zenith</td>
<td>Latitude (\varphi)</td>
<td>Origin ((o))</td>
</tr>
<tr>
<td></td>
<td>Origin ((o))</td>
<td>Origin ((o))</td>
</tr>
<tr>
<td>Normal to collector</td>
<td>(dn)</td>
<td>Tilt (\alpha)</td>
</tr>
<tr>
<td>Angle of incidence</td>
<td>(i)</td>
<td>Orientation versus south (\delta)</td>
</tr>
</tbody>
</table>

The calculation shows that \(i\) and \(h\) are related by the following equations:

\[
\sin dn = \sin \varphi \cos \alpha – \sin \alpha \cos \varphi \cos \delta \tag{2}
\]

\[
\sin an = \sin \delta \sin \alpha / \cos dn \tag{3}
\]

\[
\sin h = \sin \varphi \sin D + \cos \varphi \cos D \cos Ah \tag{4}
\]

\[
\cos i = \sin dn \sin D – \cos \varphi \cos D \cos (Ah – an) \tag{5}
\]

The hour angle is easily determined: it is 15 degrees per hour.

The value of declination \(D\) is easily obtained by (7) when you know the day of the year considered \(j\). With

\[
\Gamma = 2 \pi (j - 1) / 365 \tag{6}
\]

\[
D = (0.006918 – 0.399912 \cos \Gamma + 0.070257 \sin \Gamma – 0.006758 \cos 2\Gamma + 0.000907 \sin 2\Gamma – 0.002697 \cos 3\Gamma + 0.00148 \sin 3\Gamma) \tag{7}
\]

The value of the solar constant outside the atmosphere is given by (8).

\[
I_0 = 1367 (1.00110 + 0.034221 \cos \Gamma + 0.001280 \sin \Gamma + 0.00719 \cos 2\Gamma + 0.000077 \sin 2\Gamma) \tag{8}
\]

All these relations allow to calculate \(I_0\), \(an\), \(dn\), then \(h\) and \(I\) for any time.
Correlation (1) gives D and I knowing \( G_h \) and \( G_0 = I_0 \sin h \). Irradiation G received by the collector is then given by (9):

\[
G = I \cos i + D
\]  

(9)

It is thus possible to determine the solar radiation received, hour by hour, for a fixed solar collector, whatever its orientation \( \delta \) and tilt \( \alpha \). If it is a concentration collector, the diffuse part can be neglected, and if the collector is equipped with a sun tracking device, the values of \( \alpha \) and \( \delta \) must be recalculated at each time step (a double-tracking should ensure a normal incidence of sunlight).

References


PERRIN DE BRICHAMBAUT Ch, LAMBOLEY G. "Le rayonnement solaire au sol et ses mesures" Cahier AFEDES n° 1, 2ème édition, 1974.