ADIABATIC NOZZLE MODEL

To expand the core of Thermoptim, we add external plug-ins written in the Java language, which define both the equations and the graphical interface. These additional elements are dynamically loaded when launching the software, and appear on its screens transparent to the user, as if they were part of it. The implementation of the Thermoptim extension mechanism by adding external classes led to introduce two semantic distinctions that are important: first, that between the mono and multi-functional components (the first will involve no more than one form of energy (mechanical or thermal)), and the other one related to a new concept in the software package, that of thermocoupler. The thermocoupler are intended to complement conventional heat exchangers allowing components other than "exchange" processes to connect to one or more "exchange" process to represent a thermal coupling.

By adding external components, we can continue to use the whole Thermoptim environment, i.e. all available components and the diagram editor that allows you to very easily describe the internal structure of the system studied. Not only in such a way you significantly simplify the modeling process and facilitate subsequent use and maintenance of the model, but mostly you secure its construction by automating the establishment of linkages between its components and ensuring consistency. This is the more important that the system under study includes a large number of components.

An adiabatic nozzle is a fixed component that allows one to convert in kinetic energy the pressure of a gas.

In this note, we present a model for representing an adiabatic nozzle. After a brief reminder of the thermodynamics of the nozzle, we will present the screen of the external component defined in class Nozzle.java[1].

Thermodynamics of an adiabatic nozzle

We assume in what follows that the fluid passing through the nozzle at least locally can be considered as a perfect gas, taking a well-chosen value of its specific heat capacity c_p. The notations are those of the book Energy Systems.

The nozzle being adiabatic, we can easily show that the fluid stagnation temperature is equal to the isentropic stagnation temperature, even in the presence of irreversibilities:

$$T_{is} = T_a + \frac{C^2}{2\,c_p} \qquad (1)$$

By introducing the Mach number of flow:

$$Ma = \frac{C}{\sqrt{\gamma\, r T}} \qquad \text{and noticing that:} \qquad c_p = \frac{\gamma\, r}{\gamma - 1}$$

It comes: $\Delta T = T_{is} - T_a = \frac{C^2}{2\,c_p} = \frac{\gamma - 1}{2} Ma^2$

Equation (2.6.8) gives the isentropic stagnation pressure:

$$PT^{\gamma/(\gamma-1)} = \text{Const} \qquad \text{or} \qquad P_{is} = P_a \left(\frac{T_{is}}{T_a}\right)^{\gamma/(\gamma-1)}$$

We find:

$$P_{is} = P_a \left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{\gamma/(\gamma-1)} \qquad (2)$$

[1] http://www.thermoptim.org/sections/logiciels/thermoptim/modelotheque/modele-tuyere

Both relations (1) and (2) can be interpreted as follows: in any isentropic flow of a perfect gas in a tube with fixed walls, the stagnation temperature and pressure are conserved.

If the flow is adiabatic, but not reversible, its law is no longer an isentropic, but a polytropic. With the usual assumptions, the above relations are transformed as shown below.

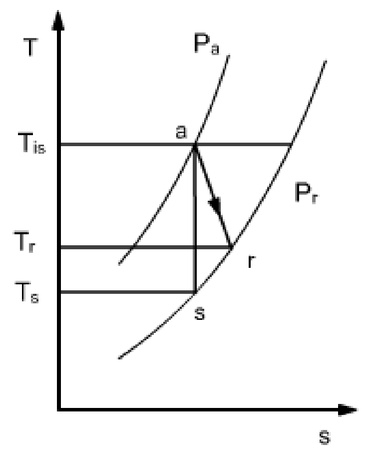

Figure 1: Nozzle process in a (T,s) chart

The gas being assumed perfect: $\Delta h = c_p \Delta T$

The total enthalpy being conserved, the stagnation polytropic and isentropic temperatures are equal:

$$T_p = T_a + \frac{C^2}{2\,c_p} = T_{is}$$

The polytropic equation gives the stagnation pressure:

$$PT^{\,k/(k-1)} = \text{Const} \qquad \text{or} \qquad P_p = P_a \left(\frac{T_p}{T_a}\right)^{k/(k-1)}$$

We find:

$$P_p = P_a \left(1 + \frac{\gamma - 1}{2}\, Ma^2\right)^{k/(k-1)} \qquad (3)$$

The polytropic stagnation pressure is not equal to the isentropic stagnation pressure, the irreversibilities resulting in losses.

In a nozzle, we know the initial and final pressures, and we can consider as a first approximation that the initial velocity is negligible.

Reasoning on the conversion of the pressure energy in velocity, the difference in inlet and outlet pressures (P_a – P_r) could theoretically provide a difference in enthalpy $\Delta h_s = h_A - h_S$ corresponding to a kinetic energy $C^2_{0a}/2$ (Figure 1). Because of irreversibilities, the exit point is R and not S, determinable if one knows the isentropic efficiency η_s:

$$\eta_s = \frac{h_A - h_R}{\Delta h_s} \qquad (4)$$

We deduce the output velocity, which gives:

$$C = \sqrt{2\,\eta_s\,|\Delta h_s|}$$

With the assumption that the gas is perfect, we can also write (4) as:

$$\eta_s = \frac{T_A - T_R}{T_A - T_S} \qquad (5)$$

Which leads to $C = \sqrt{2\,\eta_s\,c_p\,T_a \left[\left(\frac{P_r}{P_a}\right)^{(\gamma-1)/\gamma} - 1\right]}$ (6)

These relationships allow us to fully characterize the process.
In the model, we have generalized the above expressions to take into account the gas velocity existing at the inlet.

Calculation of the nozzle sections

A well-known result of fluid mechanics is that a sonic nozzle should be formed by a convergent, whose section reduces until sonic conditions are established at the throat, followed by a diverging cross-section increasing downstream of the throat. We call a Laval nozzle such convergent-divergent configuration. Since the velocity of the gas nozzle exit is supersonic, we must identify the sections at the throat and at the outlet of the nozzle.

Section S_c of the throat is given by equation (7) :

$$\frac{\dot{m}\sqrt{T_a}}{P_a} = \sqrt{\frac{\gamma}{r}}\; S_c \left[\frac{2}{\gamma + 1}\right]^{(\gamma + 1)/2(\gamma - 1)} \qquad (7)$$

The output section is calculated just knowing the speed and condition of the gas nozzle exit.

Design of the external component

General

Two calculation methods are possible: to determine the output pressure knowing the output velocity, or to determine the output velocity knowing the output pressure.

Model parameters are:

- the gas inlet velocity (m/s);
- the isentropic efficiency of the process;
- either the gas output velocity (m/s), or the gas pressure at the exit of the nozzle, depending on the calculation option chosen.

Model input data are as follows (provided by the inlet component):

- the gas temperature T_a (°C or K) at the nozzle inlet;
- the gas pressure P_a (bar) at the nozzle inlet;
- the gas flow rate $\dot{m}$ (kg/s).

The outputs are:

- either the gas pressure at the exit of the nozzle, or the gas outlet velocity (m/s), depending on the calculation option chosen;
- the gas temperature at the nozzle exit.

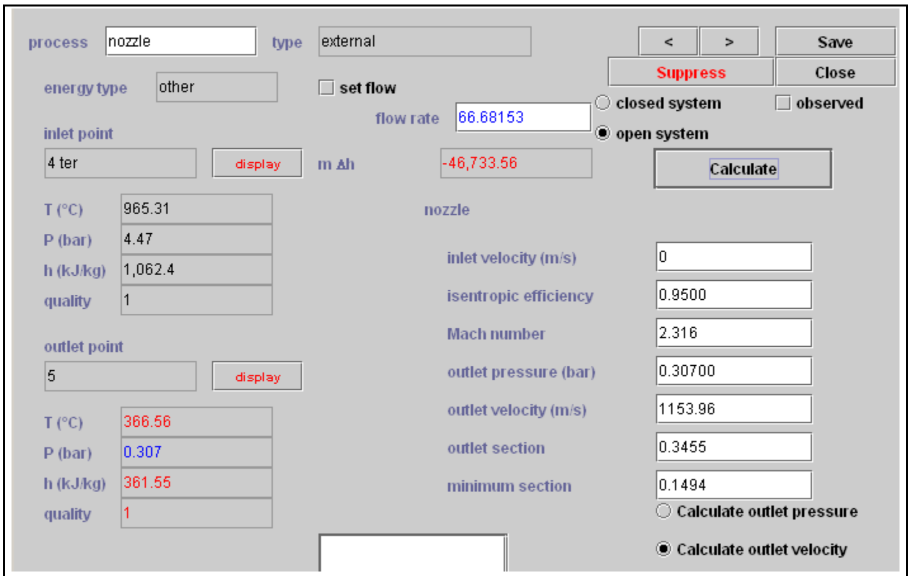

Figure 2: Nozzle screen

Graphical interface

A graphical interface for the component can be deduced (Figure 2). You have to build the bottom left of the screen, the rest being defined as a Thermoptim standard.

The input data are supplied by the inlet process of the system in which the component is inserted: gas flow and inlet point state.

Sequence of calculations

The sequence of calculations is as follows:

- update of the component before calculation with the values of the inlet process and point;
- update with the settings of the external component screen;
- calculation of the output pressure or velocity and of the outlet point state;
- update of the external component screen.

The problems encountered in practice at each of these steps are quite similar to those presented in the documentation provided in Volume 3 of Thermoptim reference manual. You should refer to it for further explanations.

Figure 3 shows how this component can be used to model a turbojet in Thermoptim.

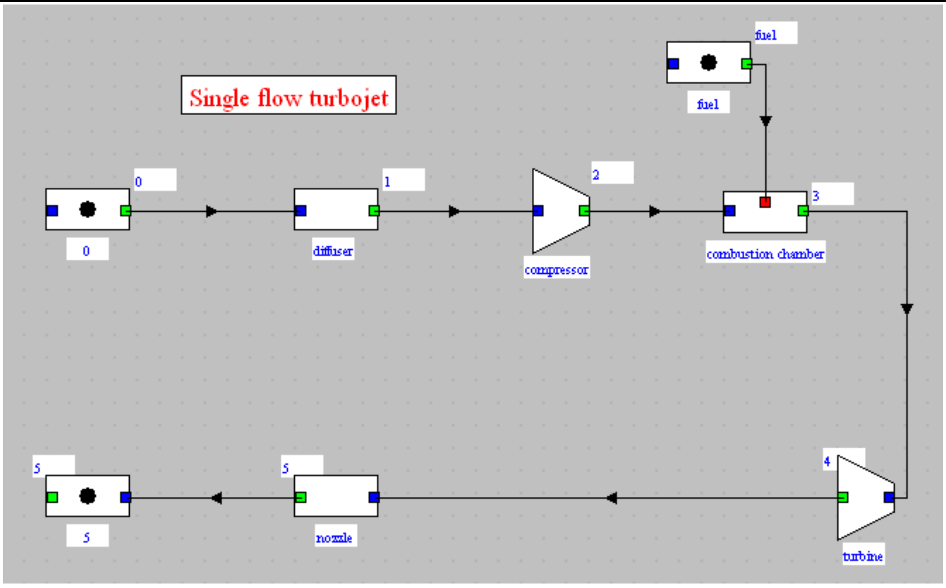

Figure 3: Thermoptim diagram of a single flow turbojet