Cumulative frequency curves of irradiation

The analysis of meteorological data required for calculations, and our experience of fine simulations led us to choose a form of data presentation that emphasizes the phenomena of thresholds and nonlinearity: **cumulative frequency curves (CFCs) of solar radiation** received by a collecting surface.

1 Curve construction

These curves (see Figure 1) are obtained as follows: one considers the historical file of hourly measurements corresponding to a given period (usually one month): solar irradiation G (averaged over one hour) may take values ranging from 0 to 1,200 W/m^2. We divide this interval into M classes of equal width (25 W/m^2 if M = 48). We then sort the hourly values to establish a table of absolute frequencies N(I), I varying from 1 to M, which gives the number of hours in the period when solar irradiation was between the bounds of class I.

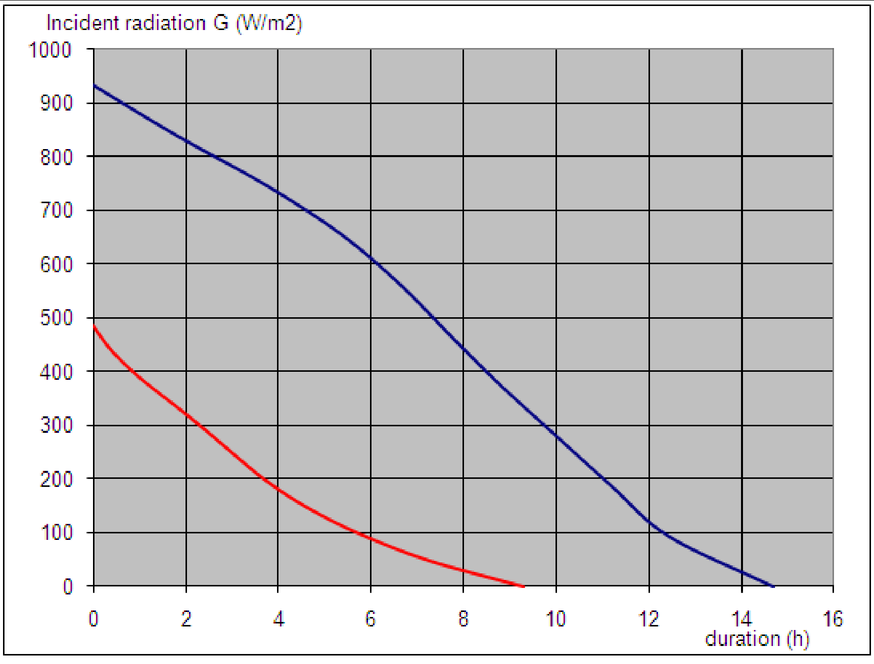

Figure 1: Ajaccio CFCs, on a horizontal plane in January and July

N_{days} being the number of days in the period, we then obtain the cumulative frequency table $n_h(I)$ by expression:

$$n_h(I) = \sum_{j=I}^{M} \frac{N(j)}{N_{days}}$$

This leads to table G(I) which contains the values of lower bounds of the various classes.

Cumulative frequency curves therefore read as follows: we have in the abscissa the number of hours n_h (reduced to one day), during which solar irradiation has exceeded the value read in the ordinate.
For example, in Figure 1, in Ajaccio in July, the threshold of 600 W/m^2 was exceeded on average 6 hours per day on a horizontal plane. The x-intercept corresponds to the average day duration for the period. The area bounded by the curve and the axes is none other than the daily average energy received over this period.

These diagrams can be plotted, of course, for different receiving surface orientations and tilts.

2 Curve smoothing

Once the cumulative frequency curve obtained, it is possible to smooth the histogram by different families of curves depending on the uses that we want to make of them.

Based on work done in the 80s at the Centre for Energy Studies of École des Mines de Paris, the best representation of these curves is explained below.

CFCs are expressed relative to reduced variables:
$x = n_h/d_d$, number of hours referred to day duration
$y = G/G_{max}$, solar radiation received referred to maximum radiation
$x = f_0(y)+A_1f_1(y)+A_2f_2(y)+A_3f_3(y)+A_4f_4(y)+A_5f_5(y)+A_6f_6(y)+A_7f_7(y)+A_8f_8(y)$

The 6 or 8 Ai, depending on the precision that we seek, can be tabulated for different locations and different inclinations.

f_i are orthogonal polynomials defined as follows:

$f_0 = (1-y)$
$f_1 = y\ (1-y)\ 30^{0.5}$
$f_2 = y\ (1-y)\ (- 1 + 2\ y)\ 210^{0.5}$
$f_3 = y\ (1-y)\ (9 - 42\ y + 42\ y^2)\ 10^{0.5}$
$f_4 = y\ (1-y)\ (- 1 + 8\ y - 18\ y^2 + 12\ y^3)\ 2310^{0.5}$
$f_5 = y\ (1-y)\ (2 - 24\ y + 90\ y^2 -132\ y^3 + 66\ y^4)\ 1365^{0.5}$
$f_6 = y\ (1-y)\ (- 18 + 300\ y - 1650\ y^2 + 3960\ y^3 - 4290\ y^4 + 1716\ y^5)\ 35^{0.5}$
$f_7 = y\ (1-y)\ (6 - 132\ y + 990\ y^2 - 3432\ y^3 + 6006\ y^4 - 5148\ y^5 + 1716\ y^6)\ 595^{0.5}$
$f_8 = y(1-y)(-6+168y-1638y^2+7644y^3-19110y^4+26208y^5-18564y^6 +5304y^5)\ 1045^{0.5}$
H being the average daily energy received, the area subtended by the CFC is:

$$\nu = \frac{H}{d_d\ G_{max}} = (0.5+A_1\ (5/6)^{0.5}+A_3\ 0.1^{0.5}+A_5\ (13/420)^{0.5}+A_7\ (17/1260)^{0.5})\ d_d\ G_{max}$$

You just need to know dj, G_{max}, H and values of the 6 Ai to reconstitute a CFC.

3 Estimation of CFCs from empirical formulas

When you do not have the values of coefficients Ai involved in the expressions giving the CFC, it is possible to reconstruct these curves from empirical correlations connecting Ai with ν:

$A_1 = -0.42297 + 0.16183\ \nu + 2.48691\ \nu^2 – 2.23697\ \nu^3$
$A_2 = 0.34623 – 1.47635\nu + 1.83694\ \nu^2$
$A_3 = -0.2789 + 1.91022\ \nu - 4.77807\ \nu^2 + 4.11137\ \nu^3$
$A_4 = 0.18865 – 1.36236\ \nu + 3.37632\ \nu^2 – 2.75771\ \nu^3$
$A_5 = -0.11845 + 0.98872\ \nu - 2.77182\ \nu^2 + 2.53303\ \nu^3$
$A_6 = 0.06968 - 0.60008\ \nu + 1.73839\ \nu^2 – 1.65775\ \nu^3$
The accuracy of such estimates being limited, 6 polynomials suffice.

You can find in the literature on solar climatology various correlations to estimate H and G_{max}, and day duration is given by $d_d = 1/w\ \mathrm{Arcos}(-\mathrm{tg}\ \varphi/\mathrm{tg}\ D)$, with w = 7.5 if the angles are expressed in degrees, and $w = 2\pi/24$ when expressed in radians.

4 Interpolation on tilt

CFC parameter values are given for 0°, 30°, 45°, 60° and 90°. For a different slope, linear interpolation is sufficient for A_i. For H and G_{max}, it is recommended to use a polynomial of degree 4 (same coefficients for H and G_{max}):

$Y_0 = H(45)$
$Y_1 = H(90) - H(0)$
$Y_2 = H(90) + H(0)$
$Y_3 = H(60) - H(30)$
$Y_4 = H(60) + H(30)$
$a = Y_0$

$$b = \frac{27\ Y_3 - Y_1}{48}$$

$$d = \frac{Y_1 - 3\ Y_3}{48}$$

$$e = \frac{16\ Y_0 + Y_2 - 9\ Y_4}{144}$$

$c = Y_4 /2 - a - e$

$$z = \frac{s - 45}{15}$$ s, inclination in degrees of the plane considered

$H(s) = a + b\ z + c\ z^2 + d\ z^3 + e\ z^4$

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